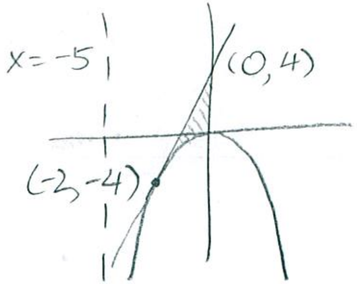
The region bounded by $y = -x^2$, $y = 4x + 4$ and $x = 0$ is revolved around the line $x = -5$. **SCORE: _____ / 35 PTS**

[a] Write, **BUT DO NOT EVALUATE**, a **single** integral for the volume of the resulting solid.



$$-x^2 = 4x+4$$
$$0 = x^2+4x+4$$
$$0 = (x+2)^2$$
$$x = -2$$

$$\underbrace{\int_{-2}^{0}}_{4} \underbrace{2\pi}_{3} \underbrace{(x+5)}_{4} \underbrace{(4x+4 - -x^2)}_{4}\, dx$$

[b] If you used the disk or washer method in [a],
write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the same volume using the shell method.
If you used the shell method in [a],
write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the same volume using the disk or washer method.

$$3\ \int_{-4}^{0} \pi\left[\left(-\overbrace{\underbrace{\sqrt{-y}}}^{4} - -5\right)^2 - \left(\frac{y-4}{4} - -5\right)^2\right] dy$$

$$+\ 3\ \int_{0}^{4} \underbrace{\pi}_{3} \left[\underbrace{(0 - -5)^2}_{2} - \underbrace{\left(\frac{y-4}{4} - -5\right)^2}_{5}\right] dy$$

$y = -x^2 \rightarrow x = \pm\sqrt{-y}$

$y = 4x+4 \rightarrow x = \frac{y-4}{4}$

A solid of revolution has volume $\int_1^3 2\pi(y^2 - \ln y)(y+4)\,dy$. Find the equation of the axis of revolution, **SCORE: _____ / 15 PTS**

and the equations of the boundaries of the region being revolved. **<u>Sketch the region being revolved.</u>**

<u>Do NOT use the x– nor y–axes as boundaries nor the axis of revolution.</u>

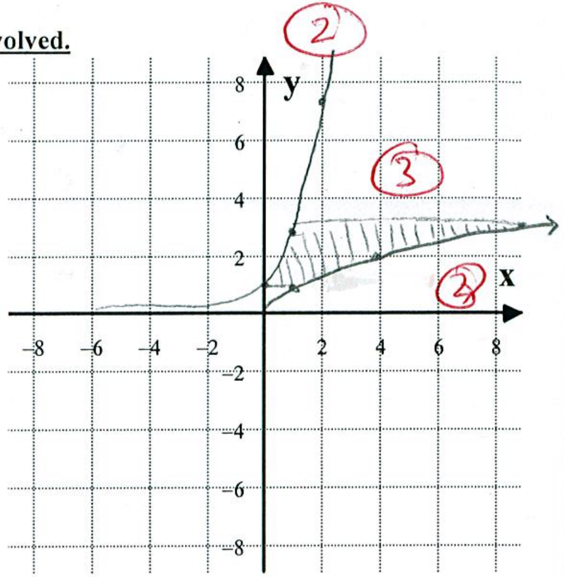
Equation of axis of revolution: $y = -4$ 2

Equations of boundaries: $y = 1$ 1

$y = 3$ 1

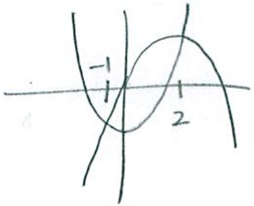
$x = y^2$ 2

$x = \ln y$ or $y = e^x$ 2



Find the area of the region between $y = 4x^2 - 12$ and $y = 6x - 2x^2$ for $-2 \le x \le 4$. **SCORE: _____ / 25 PTS**

Your final answer must be a number, not an integral.



$$4x^2 - 12 = 6x - 2x^2$$
$$6x^2 - 6x - 12 = 0$$
$$6(x^2 - x - 2) = 0$$
$$6(x+1)(x-2) = 0$$
$$x = -1, 2$$

4

$$\int_{-2}^{-1} (4x^2 - 12 - (6x - 2x^2))\,dx + \int_{-1}^{2} (6x - 2x^2 - (4x^2 - 12))\,dx + \int_{2}^{4} (4x^2 - 12 - (6x - 2x^2))\,dx$$

6

$$= \int_{-2}^{-1} (6x^2 - 6x - 12)\,dx + \int_{-1}^{2} (-6x^2 + 6x + 12)\,dx + \int_{2}^{4} (6x^2 - 6x - 12)\,dx$$

6

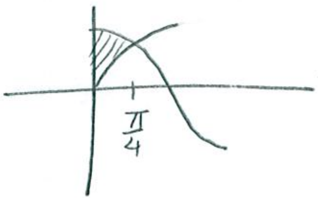
$$= (2x^3 - 3x^2 - 12x)\Big|_{-2}^{-1} + (-2x^3 + 3x^2 + 12x)\Big|_{-1}^{2} + (2x^3 - 3x^2 - 12x)\Big|_{2}^{4}$$

6

$$= 2(-1 - {}^-8) - 3(1 - 4) - 12(-1 - {}^-2) - 2(8 - {}^-1) + 3(4 - 1) + 12(2 - {}^-1)$$
$$+ 2(64 - 8) - 3(16 - 4) - 12(4 - 2) = 14 + 9 - 12 - 18 + 9 + 36 + 112 - 36 - 24$$
$$= 90$$

3

Find the **y-coordinate** of the center of mass of the region in the first quadrant bounded by $y = \cos x$, $y = \sin x$ **SCORE: _____ / 25 PTS**
and $x = 0$. **Your final answer must be a number, not an integral.**



$$\int_0^{\frac{\pi}{4}} (\cos x - \sin x)\,dx = (\sin x + \cos x)\Big|_0^{\frac{\pi}{4}} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 1$$

$$= \sqrt{2} - 1$$

$$\frac{1}{2}\int_0^{\frac{\pi}{4}} (\cos^2 x - \sin^2 x)\,dx = \frac{1}{2}\int_0^{\frac{\pi}{4}} \cos 2x\,dx$$

4

$$= \frac{1}{4}\sin 2x \Big|_0^{\frac{\pi}{4}}$$

$$= \frac{1}{4}(1) = \frac{1}{4}$$

$$\bar{y} = \frac{1}{\sqrt{2}-1} \cdot \frac{1}{4}$$

$$= \frac{\sqrt{2}+1}{4}$$

3 EXCEPT AS NOTED

At the start of each quarter, there is a 12 day window in which students may drop a course without a record of their enrolment in the class appearing on their transcript. Suppose a student is randomly selected among those who dropped a class during this quarter's 12 day window. Let X be the number of days after the start of the quarter that the student dropped their class. Suppose that the probability density function for X is given by

SCORE: ______ / 30 PTS

$$f(x) = \begin{cases} k\sin\frac{\pi x}{24}, & x \in [0,12] \\ 0, & x \notin [0,12] \end{cases}$$

[a] Find the probability that a student dropped a class during the second half of the window.

4 $\int_0^{12} k\sin\frac{\pi x}{24}\,dx = 1$

$k \cdot -\frac{24}{\pi}\cos\frac{\pi x}{24}\Big|_0^{12} = 1$ 4

$-\frac{24k}{\pi}(\cos\frac{\pi}{2} - 1) = 1$

2 $k = \frac{\pi}{24}$

5 $\frac{\pi}{24}\int_6^{12} \sin\frac{\pi x}{24}\,dx$

$= \frac{\pi}{24}\cdot -\frac{24}{\pi}\cos\frac{\pi x}{24}\Big|_6^{12}$

$= -(\cos\frac{\pi}{2} - \cos\frac{\pi}{4})$

5 $= \frac{\sqrt{2}}{2} \approx 0.7071$ OR 71%

[b] Find the median number of days before students drop their classes.

$\frac{\pi}{24}\int_0^{m} \sin\frac{\pi x}{24}\,dx = \frac{1}{2}$ 4

$\frac{\pi}{24}\cdot -\frac{24}{\pi}\cos\frac{\pi x}{24}\Big|_0^{m} = \frac{1}{2}$

$\cos\frac{\pi m}{24} - 1 = -\frac{1}{2}$ 4

$\cos\frac{\pi m}{24} = \frac{1}{2}$

$\frac{\pi m}{24} = \frac{\pi}{3}$

2 $m = 8$ DAYS

Find the length of the curve $y = \int_{-1}^{x} \sqrt{e^{4t}(e^{4t}+2)}\, dt$ for $0 \le x \le 2$. SCORE: _____ / 20 PTS

Your final answer must be a number, not an integral.

$$\frac{dy}{dx} = \sqrt{e^{4x}(e^{4x}+2)} \text{ BY FTC PART 1}$$

$$\int_0^2 \sqrt{1 + \left(\sqrt{e^{4x}(e^{4x}+2)}\right)^2}\, dx$$

$$= \int_0^2 \sqrt{e^{8x} + 2e^{4x} + 1}\, dx$$

4½ EACH EXCEPT AS NOTED

$$= \int_0^2 (e^{4x} + 1)\, dx$$

2

$$= \left(\frac{1}{4}e^{4x} + x\right)\Big|_0^2 = \frac{1}{4}(e^8 - 1) + 2 = \frac{1}{4}e^8 + \frac{7}{4}$$